PHYSICS				CHEMISTRY				MATHS			
1	b	**14**	a	**26**	a	**39**	b	**51**	b	**64**	d
2	d	**15**	d	**27**	d	**40**	c	**52**	c	**65**	d
3	a	**16**	c	**28**	d	**41**	a	**53**	a	**66**	b
4	a	**17**	a	**29**	c	**42**	d	**54**	d	**67**	a
5	d	**18**	a	**30**	a	**43**	d	**55**	a	**68**	a
6	b	**19**	a	**31**	d	**44**	d	**56**	b	**69**	d
7	c	**20**	c	**32**	c	**45**	c	**57**	a	**70**	b
8	d	**21**	6	**33**	a	**46**	5	**58**	c	**71**	1
9	b	**22**	363	**34**	a	**47**	6	**59**	a	**72**	18
10	a	**23**	300	**35**	b	**48**	4	**60**	b	**73**	8
11	c	**24**	20	**36**	d	**49**	6	**61**	c	**74**	81
12	b	**25**	6	**37**	a	**50**	3	**62**	c	**75**	2
13	d			**38**	a			**63**	d		

HINTS

1.
(b) Since for such liquid (Non-wetting) angle of contact is obtuse.

When a non-wetting liquid is placed in a capillary tube, the liquid molecules have a stronger attraction to each other (cohesive forces) than to the surface of the tube (adhesive forces). This results in the liquid forming a convex meniscus, curving upwards within the tube.

Why other options are incorrect:

a) Concave:

A concave meniscus occurs when the liquid wets the tube, meaning the liquid molecules have a stronger attraction to the tube surface than to each other. This causes the liquid to curve downwards. 🔗

c) Flat:

A flat meniscus would occur if the liquid had equal attraction to the tube surface and its own molecules, which is not the case for a non-wetting liquid. 🔗

d) Any of these depending on the radius of the tube:

The radius of the tube affects the height of capillary rise, but not the shape of the meniscus. The shape is determined solely by the liquid's wetting properties relative to the tube material. Wetting liquids form concave menisci, while non-wetting liquids form convex menisci. 🔗

2.
d) Propelling force provided to an aeroplane by its propellers

Why: a & b are classic Bernoulli applications (pressure decreases where speed is higher). Propeller thrust is mainly from momentum change of air (Newton's 3rd law), not Bernoulli.

Bernoulli's principle is about the relationship between fluid speed and pressure, where higher speed

leads to lower pressure. The lift of an airplane wing, the operation of a venturimeter, and the effect of blowing over paper are all based on this principle. However, the propelling force of an airplane propeller is primarily due to its design and interaction with air, not the pressure difference caused by fluid speed.

3	(a) Energy released $= (A_f - A_i)T$

$$A_f = 4\pi R^2 = \frac{3}{3}4\pi\frac{R^3}{R} = \frac{3V}{R}$$

$$A_i = n \times 4\pi r^2 = \frac{V}{\frac{4}{3}\pi r^3}4\pi r^2 = \frac{3V}{r}$$

$$\Rightarrow \text{Energy released} = 3VT\left[\frac{1}{r} - \frac{1}{R}\right].$$

4.	(a) Rise of a liquid in a capillary tube, $h = \dfrac{2S\cos\theta}{r\rho g}$

Here, $r = 0.07\,cm = 0.07 \times 10^{-2}m$

For water, $S = 0.07\,Nm^{-1}$, $\rho = 10^3\,kgm^{-3}$

Angle of contact $\theta = 0°$

$$\therefore h = \frac{2 \times (0.07\,Nm^{-1}) \times 1}{(0.07 \times 10^{-2}\,m)(10^3\,kgm^{-3})(10\,ms^{-2})}$$

$$= 2 \times 10^{-2}\,m = 2\,cm.$$

5.	(d) Excess pressure inside the soap bubble $= 2\,T/R$

If $P_1 = \dfrac{2T}{R}$; $P_2 = \dfrac{2T}{R'}$

i.e. $P_1 = \dfrac{2T}{R} = 2 \times \dfrac{2T}{R'} \Rightarrow \dfrac{2T}{R} = \dfrac{2T}{R'/2}$

$\therefore R = R'/2 \Rightarrow R' = 2R$

$$m_1 = \frac{4}{3}\pi R^3\rho; \quad m_2 = \frac{4}{3}\pi(2R)^3\rho = 8m_1$$

$\therefore m_1 = n.m_2$ [Given]

$\dfrac{m_2}{8} = n.m_2 \Rightarrow n = 1/8 = 0.125.$

6.	(b) Velocity of efflux when the hole is at depth h, $v = \sqrt{2gh}$

Rate of flow of water from square hole

$Q_1 = a_1v_1 = L^2\sqrt{2gy}$

Rate of flow of water from circular hole

$Q_2 = a_2v_2 = \pi R^2\sqrt{2g(4y)}$

According to problem $Q_1 = Q_2$

$\Rightarrow L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)} \Rightarrow R = \dfrac{L}{\sqrt{2\pi}}.$

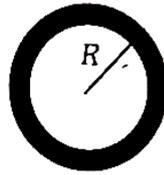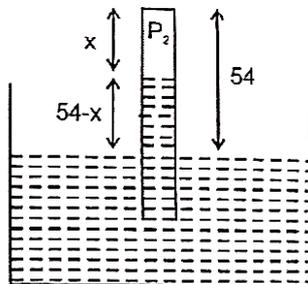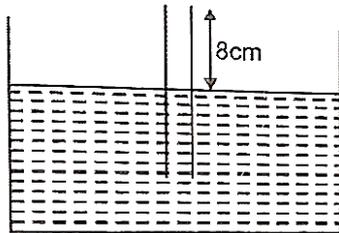| 7.	(c) Here, $\eta = 10^{-3}\,Nm^{-2}s$, $v = 5\,ms^{-1}$; $l = 10\,m$
	Strain rate $= \dfrac{v}{l}$
	Coefficient of viscosity, $\eta = \dfrac{\text{Shearing strees}}{\text{Strain rate}}$
	$\therefore$ Shearing strees $= \eta \times$ Strain rate
	$= \dfrac{(10^{-3}\,Nm^{-2}s)(5\,ms^{-1})}{(10\,m)} = 0.5 \times 10^{-3}\,Nm^{-2}.$
8.	(d) Diameter $= 8 \times 10^{-3}\,m$
	$v_1 = 0.4\,m/s$
	$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2 \times 10 \times 0.2} = 2\,m/s$
	$A_1 v_1 = A_2 v_2$
	$\pi\left(\dfrac{8 \times 10^{-3}}{2}\right)^2 \times 0.4 = \pi \times \dfrac{d^2}{4} \times 2 \Rightarrow d \approx 3.6 \times 10^{-3}\,m.$
9.	(b) Assuming ρ to be specific gravity,
	$(4\pi R^2 t)\rho \times g \leq \dfrac{4}{3}\pi R^3 \times 1 \times g.$
	Therefore $t \leq \dfrac{R}{3\rho}.$
10.	(a)
	$(76)(8) = (54 - x)(76 - x) \Rightarrow x = 38cm$
	Length of air column $= 54 - 38 = 16cm.$

Initially: air column = 8 cm at $P_1 = 76$ cm Hg.

After sealing and lifting tube up by 46 cm:

Let final air length = L and mercury level difference inside vs outside = H.
Then

$$P_2 = 76 - H, \qquad 76 \times 8 = (76 - H)L \quad \text{(Boyle)}$$

Geometry (top moved up 46 cm):

$$L + H = 8 + 46 = 54.$$

Solving gives $L = 16$ cm, $H = 38$ cm.

Answer: (a) 16 cm

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11. (c) Let the total volume of ice-berg is V and its density is ρ.

If this ice-berg floats in water with volume V_{in} inside it

then $V_{in}\sigma g = V\rho g \Rightarrow V_{in} = \left(\dfrac{\rho}{\sigma}\right)V$ [σ = density of water]

$\Rightarrow V_{out} = V - V_{in} = \left(\dfrac{\sigma - \rho}{\sigma}\right)V$

$\Rightarrow \dfrac{V_{out}}{V} = \left(\dfrac{\sigma - \rho}{\sigma}\right) = \dfrac{1000 - 900}{1000} = \dfrac{1}{10}$

$\therefore V_{out} = 10\%$ of V.

---

12. (b) Rate of flow of liquid $V = \dfrac{P}{R}$

where liquid resistance $R = \dfrac{8\eta l}{\pi r^4}$

For another tube liquid resistance

$R' = \dfrac{8\eta l}{\pi \left(\dfrac{r}{2}\right)^4} = \dfrac{8\eta l}{\pi r^4}.16 = 16R$

For the series combination

$V_{New} = \dfrac{P}{R + R'} = \dfrac{P}{R + 16R} = \dfrac{P}{17R} = \dfrac{V}{17}.$

13	(d) The terminal velocity of the spherical body of radius R, density ρ falling through a liquid of density σ is given by

$$v_T = \frac{2}{9}\frac{R^2(\rho - \sigma)g}{\eta}$$

where η is the coefficient of viscosity of the liquid

$$\therefore \ v_{T_1} = \frac{2R_1^2(\sigma_1 - \sigma)g}{9\eta} \ \text{ and } \ v_{T_2} = \frac{2R_2^2(\sigma_2 - \sigma)g}{9\eta}$$

According to the given problem, $v_{T_1} = v_{T_2}$

$$R_1^2(\rho_1 - \sigma) = R_2^2(\rho_2 - \sigma) \ \text{ or } \ \frac{R_1^2}{R_2^2} = \frac{\rho_2 - \sigma}{\rho_1 - \sigma}$$

Substituting the given values, we get

$$\frac{R_1^2}{R_2^2} = \frac{(11 \times 10^3 - 2 \times 10^3)}{(8 \times 10^3 - 2 \times 10^3)} = \frac{9}{6} = \frac{3}{2}$$

$$\frac{R_1}{R_2} = \sqrt{\frac{3}{2}}.$$

14.

(a) $P + \dfrac{1}{2}\rho v^2 = \text{constant}$

$$\Rightarrow P + \frac{1}{2}\rho v^2 = P' + \frac{1}{2}\rho(2v)^2 \Rightarrow P' = P - \frac{3}{2}\rho v^2.$$

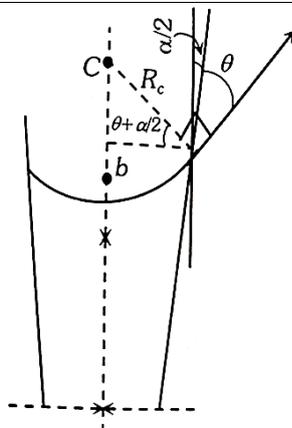
15. (d) Using geometry

$$\frac{b}{R_e} = \cos\left(\theta + \frac{\alpha}{2}\right)$$

Using Pressure method

$$P_0 - \frac{2S}{R_c} + h\rho g = P_0$$

$$\Rightarrow h = \frac{2S}{R_e \rho g}$$

$$= \frac{2S}{b\rho g}\cos(\theta + \alpha/2).$$

OR

Capillary rise in a truncated cone (frustum)

Let the cone's **apex angle** = $\alpha \Rightarrow$ semi-vertical angle = $\beta = \alpha/2$.

At height h the tube radius is b. For a spherical meniscus of radius R,

$$\rho g h = \frac{2S}{R} \quad \text{(pressure balance)}.$$

Geometry at the contact line: the angle between the meniscus radius and the wall is $90° - \theta$; since the wall makes angle β with the axis,

$$\angle(\text{radius, axis}) = (90° - \theta) - \beta$$

$$\Rightarrow b = R\sin\big((90° - \theta) - \beta\big) = R\cos(\theta + \beta).$$

Hence $R = \dfrac{b}{\cos(\theta + \beta)}$, giving

$$\boxed{h = \frac{2S}{\rho g\, b}\cos\left(\theta + \frac{\alpha}{2}\right).}$$

NB.

For a truncated cone with an apex angle α, when the liquid rises to a height where the cross-section has radius b, the effective contact angle for the meniscus formed will be affected by the cone's angle. The angle that the meniscus makes with the vertical (or the angle of the tube wall with the vertical) needs to be considered.

In the case of a truncated cone with apex angle α, the angle of the tube wall with the vertical axis is $\alpha/2$. When the liquid rises, the meniscus curvature effectively relates to the radius b and the combined angle of the contact angle θ and the cone's half-apex angle $\alpha/2$. Thus, the effective angle in the formula becomes $\left(\theta + \frac{\alpha}{2}\right)$ if measured with respect to the horizontal plane of the meniscus, or in the formula it's usually written as $\cos\left(\theta - \frac{\alpha}{2}\right)$ or $\cos\left(\theta + \frac{\alpha}{2}\right)$ depending on the exact definition of θ and the geometry. &

Upon examining similar solved problems, the modification to the contact angle for a truncated cone is typically given as $\left(\theta + \frac{\alpha}{2}\right)$ when considering the angle between the tangent to the liquid surface and the wall of the capillary at the point of contact, which is then used in the Jurin's law formula. &

The radius of curvature R of the meniscus can be related to the radius of the tube at that height b and the modified contact angle. From the geometry, $b = R\cos\left(\theta + \frac{\alpha}{2}\right)$.
Therefore, $R = \dfrac{b}{\cos\left(\theta + \frac{\alpha}{2}\right)}$. &

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16. (c) At terminal speed

$a = 0,\ F_{net} = 0$

$mg = F_v = 6\pi\eta Rv$

$v = \dfrac{mg}{6\pi\eta Rv}$

$v = \dfrac{\rho_w \dfrac{4\pi}{3} R^3 g}{6\pi\eta R} = \dfrac{2\rho_w R^2 g}{9\eta} = \dfrac{400}{81}\,m/s = 4.94\,m/s.$

Neglect buoyancy $\Rightarrow$ use $\rho \approx \rho_w = 1000\,\text{kg m}^{-3}$.

$v_t = \dfrac{2}{9}\dfrac{R^2(\rho_w - \rho_a)g}{\eta} \approx \dfrac{2}{9}\dfrac{(2\times10^{-4})^2(1000)(10)}{1.8\times10^{-5}} = 4.94\,\text{m s}^{-1}.$

17.	(a) $P_1 = \rho g d + P_0 = 3 \times 10^5 \, Pa$
	$\therefore \rho g d = 2 \times 10^5 \, Pa$
	$P_2 = 2\rho g d + P_0$
	$= 4 \times 10^5 + 10^5 = 5 \times 10^5 \, Pa$
	% increase $= \dfrac{P_2 - P_1}{P_1} \times 100$
	$= \dfrac{5 \times 10^5 - 3 \times 10^5}{3 \times 10^5} \times 100 = \dfrac{200}{3} \%.$
18.	a) In steady flow of incompressible liquid rate of flow remains constant i.e., V = av = const. This is equation of continuity. When pipe is placed vertically upward velocity of flow decreases with height so area of cross section increases and when pipe is placed vertically downward velocity of flow increases in downward direction so area of cross section decreases i.e., it becomes narrower. Upward jet: gravity **slows** the water $(v \downarrow) \Rightarrow$ by continuity $Q = Av = \text{const}$, the cross-sectional area **increases** $\Rightarrow$ jet **spreads**. Downward jet: gravity **speeds up** the water $(v \uparrow) \Rightarrow A \downarrow \Rightarrow$ jet **narrows**. So the constancy of volume flow rate in steady, incompressible flow (Stmt-2) directly explains Stmt-1.
19.	Surface energy $= T.\Delta A$ Where T is the surface tension of liquid and ΔA is the change in its surface area. **Answer: a) Statement 1 is true, statement 2 is true.** **Why (concise):** Surface molecules have fewer neighbours → higher potential energy → **surface energy**. Since $U = T \times \text{area}$, a drop tends to **decrease its surface area** to **minimize energy**.
20.	**A→ q; B→ p; C→ s; D→ r** Capillaries of smaller radii show rise of liquid level due to concave meniscus. Angle of contact is acute for concave meniscus, obtuse for convex and zero for flat meniscus. Angle of contact = 90° (obtuse) for mercury and glass pair. For mercury and glass pair the meniscus of mercury will be downward concave.

Column I	Column II	Explanation
A) Capillaries of smaller radii	(q) Greater height acquired by liquid in capillary tube	Smaller radius → higher rise ($h \propto 1/r$).
B) Angle of contact = 90°	(p) Flat meniscus	At 90°, no curvature = flat.
C) Contact angle > 90°	(r) Meniscus is downward concave	For $\theta > 90°$ (e.g., mercury), meniscus convex.
D) Mercury in glass tube	(s) For mercury and glass pair	Special case, mercury forms convex meniscus with glass.

21.

Solution $R = \sqrt{2gh} \times \sqrt{\dfrac{(12-h) \times 2}{g}}$

$\sqrt{4h(12-h)} = R$

For maximum R

$\dfrac{dR}{dh} = 0$

$\Rightarrow h = 6\,m$

$= 6$ **Ans.**

12 m, h, $v = \sqrt{2gh}$, $(12{-}h)$

22. **Solution** From continuity equation

$av_1 = \dfrac{a}{2} v_2$

$v_2 = 2v_1$

From Bernoulli's theorem,

$P_1 + \rho g h_1 + \dfrac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \dfrac{1}{2}\rho v_2^2$

$P_1 - P_2 = \rho\left[\left(\dfrac{v_2^2 - v_1^2}{2}\right) + g(h_2 - h_1)\right]$

$4100 = 800\left[\left(\dfrac{4v_1^2 - v_1^2}{2}\right) + 10 \times (0-1)\right]$

$\dfrac{41}{8} + 10 = \dfrac{3v_1^2}{2} \Rightarrow \dfrac{121}{8} \times \dfrac{2}{3} = v_1^2$

$v_1 = \sqrt{\dfrac{121}{4 \times 3} \times \dfrac{3}{3}} \Rightarrow v_1 = \dfrac{\sqrt{363}}{6} \, m/s$

$x = 363$ **Ans.**
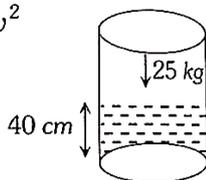
23.	**Solution** By Bernoulli's theorem

$$P_0 + \frac{250}{0.5} + \rho g(40 \times 10^{-2}) = P_0 + \frac{1}{2}\rho v^2$$

$$500 + \frac{1000 \times 10 \times 40}{100} = \frac{1}{2} \times 1000 \times v^2$$

$$v = 3\,m/s$$

$$v = 300\,cm/s$$

$$= \mathbf{300\ Ans.}$$

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24. **Solution** Speed after falling through height h should be equal to

terminal velocity $\sqrt{2gh} = \frac{2}{9}\frac{r^2(d-\rho)g}{\eta}$

$$\sqrt{2gh} = \frac{2}{9}\frac{10^{-8}(10000-1000)\times 10}{10^{-5}}$$

$$= \frac{2}{9}\times 10^{-8}\frac{9\times 10^4}{10^{-5}} = 20$$

$$2\times 10 \times h = 400 \Rightarrow h = 20\,m = \mathbf{20\ Ans.}$$

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25. **Solution** Excess pressure inside the soap bubble $= \frac{4S}{r}$

So the pressure inside the soap bubble $= P_{atm} + \frac{4S}{r}$

From ideal gas equation $PV = nRT$

$$\frac{P_A V_A}{P_B V_B} = \frac{n_A}{n_B} \Rightarrow \frac{\left(8 + \frac{4S}{r_A}\right)\frac{4}{3}\pi(r_A)^3}{\left(8 + \frac{4S}{r_B}\right)\frac{4}{3}\pi((r_B)^3)} = \frac{n_A}{n_B}$$

Substituting $S = 0.04\,N/m$, $r_A = 2\,cm$, $r_B = 4\,cm$.

$$\frac{n_A}{n_B} = \frac{1}{6}$$

$$\therefore \frac{n_B}{n_A} = \mathbf{6\ Ans.}$$


26

Aluminium chloride (AlCl₃) in acidified aqueous solution:

- In aqueous solution, AlCl₃ hydrolyzes, and the aluminum ion tends to form hydrated complex ions.
- Typically, Al^{3+} coordinates with six water molecules, forming the hexaaquaaluminum(III) ion $[Al(H_2O)_6]^{3+}$.
- This complex ion has an **octahedral** geometry because 6 ligands arrange themselves symmetrically around the central metal ion.

So the geometry is **octahedral**.
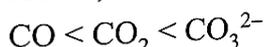
Answer: a) Octahedral

| 27 | **(d)** Structures of CO, CO$_2$ and CO$_3^{2-}$ are: |

$$C \xLeftarrow{} \ddot{O} \qquad O = C = O$$
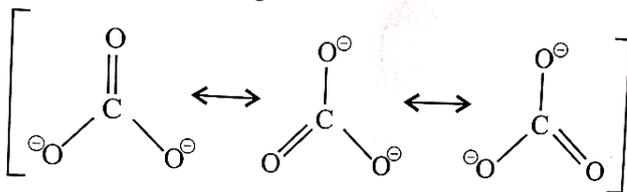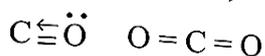
Compound	CO	CO$_2$	CO$_3$
Bond order	3	2	1.33

$$\text{Bond order} \propto \frac{1}{\text{Bond length}}$$

Hence, the decreasing (C — O) bond length is:

$$CO < CO_2 < CO_3^{2-}$$

| 28 | Both Si and B are in the same group 13 of the periodic table (though Si is actually in group 14, so this is tricky; let's clarify the usual reasoning). |

Actually:

- Silicon (Si) is in group 14.
- Boron (B) is in group 13.

But Si often shows some chemical similarity with boron because of similar electronegativity (EN) and chemical behavior in forming covalent bonds.

Checking each option:

a) Similar values of atomic radii?

- Atomic radius of B ≈ 0.87 Å
- Atomic radius of Si ≈ 1.11 Å
  They are not very close, so this is not strongly similar.

b) Similar values of electronegativity (EN)?

- EN of B ≈ 2.04 (Pauling scale)
- EN of Si ≈ 1.90
  These are relatively close, indicating similarity in bonding behavior.

c) Similar value of Ionization Energy (IE)?

- IE of B ≈ 8.3 eV
- IE of Si ≈ 8.15 eV
  These are quite close, indicating similar ease of losing electrons.

Conclusion:

- Both electronegativity (EN) and ionization energy (IE) are quite similar.
- Atomic radius is less similar.

So, the best answer is:d) All of the above (since some atomic radius similarity exists, plus EN and IE

similarities).

Answer: d) All of the above

| 29 | Graphite can be converted into diamond only at very high temperatures and pressures. |

Let's analyze the given equilibrium reaction and the conditions affecting it:

$$C_{(s)}\text{diamond} \rightleftharpoons C_{(s)}\text{graphite} + \text{heat}$$

- ΔH = -1.9 kJ/mole, meaning the reaction is exothermic in the forward direction (diamond to graphite).
- Density of diamond = 3.5 g/cm³
- Density of graphite = 2.3 g/cm³

Understanding the problem:

1. Effect of temperature:
   o Since the forward reaction is exothermic (releases heat), increasing the temperature will favor the reverse reaction (Le Chatelier's Principle), shifting equilibrium toward diamond.
   o Decreasing temperature favors the forward reaction (toward graphite).
2. Effect of pressure:
   o Higher density means lower volume for the same mass.
   o Diamond has higher density (3.5 g/cm³) than graphite (2.3 g/cm³), so diamond occupies less volume than graphite.
   o Increasing pressure favors the side with lower volume, which is diamond.
   o Decreasing pressure favors graphite.

To shift the equilibrium to the left (towards diamond):

- Low temperature favors the forward reaction (towards graphite), so not good.
- High temperature favors diamond (left), good.
- High pressure favors diamond (left), good.
- Low pressure favors graphite (right), not good.

Thus, the equilibrium shifts to the left (diamond) at high temperature and high pressure.

Correct answer: c) High temperature and high pressure

| 30 | (a) Due to the inert pair effect, thallium exists in more than one oxidation state. Also, for thallium + 1 oxidation state is more stable than +3 oxidation state. |

Explanation:

- Aluminium (Al) is usually found in the +3 oxidation state.
- Thallium (Tl), a heavier group 13 element, shows both +1 and +3 oxidation states.

Why?

The reason is the Inert Pair Effect:

- The inert pair effect is the tendency of the outermost s-electrons (the "inert pair") to remain non-ionized or unshared in heavier elements, especially in groups 13-16.
- In heavier elements like Thallium, the 6s electrons are held more tightly due to poor shielding

by the d and f electrons.
- This makes the +1 oxidation state (where the s-electrons remain paired and un-ionized) more stable than +3.
- In lighter elements like Aluminium, the +3 state is more common because the s-electrons are easily ionized.

Other options:

- Diagonal relationship: Similarity between diagonally adjacent elements in the periodic table; not relevant here.
- Lattice effect: Concerns lattice energy and solid-state stability.
- Lanthanide contraction: Related to decrease in atomic size across lanthanides, affecting elements beyond, but not directly the oxidation states of Tl or Al.

Correct answer: a) Inert pair effect

31	**(d)** Carbon-carbon bond length is maximum in diamond because diamond has all single bonds while graphite, C_{70} and C_{60} have single and double bonds.

Options:

- Graphite: Carbon atoms arranged in planar sheets with sp² hybridization. The C–C bond length is about 0.142 nm.
- C_{70}: A fullerene molecule, similar to C60 but elongated. Fullerenes have curved structures with sp² hybridized carbon atoms, bond lengths typically around 0.139–0.145 nm.
- C_{60}: Buckminsterfullerene, spherical molecule made of 60 carbon atoms, also with sp² hybridization and bond lengths around 0.139–0.145 nm.
- Diamond: Carbon atoms arranged in a 3D tetrahedral lattice with sp³ hybridization. The C–C bond length here is longer than in graphite, about 0.154 nm.

Conclusion:

- The C–C bond length is longest in diamond due to the tetrahedral sp³ bonds.
- Graphite and fullerenes (C_{60} and C_{70}) have shorter bond lengths due to sp² bonding.

Correct answer:
d) Diamond

32 The nature of electrons in graphite:

- Each carbon atom in graphite is sp² hybridized.
- Three sp² orbitals form strong σ-bonds with three neighboring carbon atoms in the plane.
- The fourth valence electron is in an unhybridized p-orbital, perpendicular to the plane.
- These p-orbitals overlap side-by-side across the lattice, forming a delocalized π-electron cloud that is spread over the entire layer.
- These delocalized electrons are not localized on individual atoms or specific bonds but are spread out over the graphene layers, allowing electrical conductivity.

So, the electrons are:

- Not localized on each carbon atom (eliminates option a)
- Not localized on every third carbon atom (no such localization)
- Not in antibonding orbitals (they are in bonding/delocalized π orbitals)
- They are delocalized (spread out) over the structure

	Answer: c) Spread out between the structure
33	**Ans. (a)**

B_2H_6 is electron deficient molecule because boron atom has three half-filled orbitals in excited state. The structure of B_2H_6 is represented as follows:
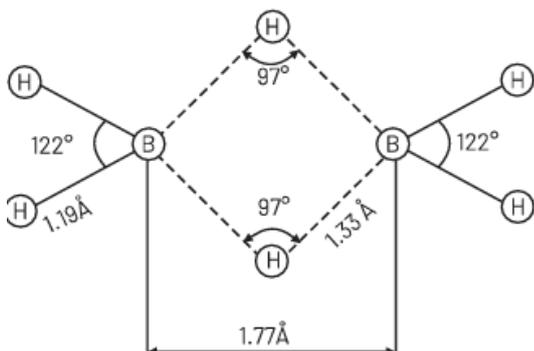
In it two electrons of a B—H bond are involved in formation of three centre bond, these bonds are represented as dotted lines. |
| 34 | **Ans. (a)**

Boron belongs to 2nd period of the periodic table with electronic configuration $1s^2, 2s^2 2p^1$. It does not have vacant d-orbitals, thus cannot increase its covalency above four.
Therefore, boron (B) cannot form MF_6^{3-} ion. In contrast, aluminium (Al), gallium (Ga), indium (In) have the vacant $3d$-orbitals, thus can increase their covalence above four and form MF_6^{3-} ion. |
| 35 | **(b)** Germanium, silicon and copper oxide are semi conductor while graphite is a conductor. |
| 36 | **(d)** Statement I is true but statement II is false.

Carbon is able to show allotropic forms due to property of catenation and $p\pi - p\pi$ bond formation.

Statement (I): SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.

- SiO_2 and GeO_2 are indeed acidic oxides.
- SnO and PbO are well-known amphoteric oxides.

So, Statement (I) is true.

Statement (II): Allotrophic forms of carbon are due to property of catenation and $p\pi$ - $d\pi$ bond formation.

- Carbon exhibits catenation, meaning it can form strong C–C bonds leading to various allotropes (diamond, graphite, graphene, fullerenes).
- The $p\pi$ - $d\pi$ bond formation is not relevant for carbon allotropes since carbon has no accessible d orbitals. |

	• So, this part of the statement is incorrect for carbon. Thus, Statement (II) is false. Final Conclusion: • Statement I: True • Statement II: False Correct option: d) Statement I is true but Statement II is false
37	Bond strengths (approximate bond dissociation energies in kJ/mol): • Si–Si: ~226 kJ/mol • Si–O: ~452 kJ/mol • Si–Cl: ~381 kJ/mol • Si–H: ~318 kJ/mol Analysis: • Si–O bond is the strongest due to strong covalent bonding and electronegativity difference. • Si–Cl is stronger than Si–H but weaker than Si–O. • Si–H bond is stronger than Si–Si. • Si–Si bond is the weakest because it is a single bond between two relatively large atoms with less overlap. Answer: a) Si – Si is the weakest bond.
38	[**Hint :** CCl_4 does not form hexachloro complex as *d*-orbitals are not present in carbon but rest of the tetrahalides can form hexahalo complexes.] Lewis acid behavior: • A Lewis acid accepts an electron pair. • Tetrahalides of group 14 elements can act as Lewis acids by accepting electron pairs because the central atom can expand its octet or has vacant orbitals. Tetrahalides given: a) CCl_4 (carbon tetrachloride) b) SiF_4 (silicon tetrafluoride) c) $GeCl_4$ (germanium tetrachloride) d) $SnCl_4$ (tin tetrachloride) Analysis: • CCl_4: Carbon is small, and the 2p orbitals do not expand the octet easily; CCl_4 is very stable and does not act as a Lewis acid. • SiF_4, $GeCl_4$, and $SnCl_4$: Heavier elements from group 14 with available d orbitals can accept electron pairs, acting as Lewis acids. Correct answer: a) CCl_4

39	**(b)** $\%$ of p- in graphite $(sp^2) = \dfrac{2}{3} \times 100 = 67\%$
	$\%$ of p- in diamond $(sp^3) = \dfrac{3}{4} \times 100 = 75\%$

40	**(c)** Melting point: $B > Al > Tl > In > Ga$
	Ionic radius $(M^{+3}/pm(: Tl > In > Ga > Al < B$
	$(\Delta_{IE}H)_1 \left(\dfrac{kJ}{mol} \right) B > Tl > Al \approx Ga > In$
	Atomic radius (in pm): $Tl > In > Al > Ga > B$

41	**(a)** Buckminster fullerene is a type of fullerene with the formula C_{60}. It has a cage like fused ring structure that resembles a soccer ball, made of 20 hexagons and 12 pentagons. Each carbon atom has three bonds.

Structure of C_{60}:

- C_{60} is a spherical molecule made of 60 carbon atoms.
- It resembles a soccer ball, consisting of:
  - 12 five-membered rings
  - 20 six-membered rings
- Each carbon is sp² hybridized and forms three sigma (σ) bonds.
- The five-membered rings are arranged so they are only adjacent to six-membered rings.
- Six-membered rings are adjacent to both six-membered and five-membered rings.

Let's evaluate each option:

a) It contains 12 six–membered rings and 24 five–membered rings, Incorrect

- Actually, C_{60} contains 12 five-membered rings and 20 six-membered rings.
- So this statement is wrong.

b) The six-membered rings are fused to both six and five-membered rings, Correct

- This is true based on the structure.

c) Each carbon atom forms three sigma bonds, Correct

- All carbons are sp² hybridized forming three σ bonds.

d) The five-membered rings are fused only to six-membered rings, Correct

- No two five-membered rings are adjacent; they are surrounded by six-membered rings.

Correct answer (i.e., incorrect statement):
a) It contains 12 six–membered rings and 24 five–membered rings

42	**(d)** GeO - Acidic $\quad$ GeO_2 - Acidic

(d) GeO - Acidic $\qquad$ GeO_2 - Acidic

SiO_2 - Acidic $\qquad$ CO - Neutral

SnO_2 - Amphoteric $\qquad$ PbO_2 - Amphoteric

Options analysis:

- a) GeO, CO — CO is acidic, not amphoteric. So **NO**.
- b) SiO_2, GeO_2 — SiO_2 is acidic, GeO_2 is weakly amphoteric. So **NO** (SiO_2 excluded).
- c) SnO_2, CO — CO is acidic, so **NO**.
- d) SnO_2, PbO_2 — both amphoteric, so **YES**.

Correct answer: **d) SnO_2, PbO_2**

43	**(d)** Reluctance of valence shell electrons to participate in bonding is called inert pair effect. The stability of lower oxidation state (+2 for group 14 elements) increases on going down the group. So the correct order is: $$SiX_2 < GeX_2 < SnX_2 < PbX_2$$

44	**(d)** **Statement I :** Number of d and f electrons, increases down the group and due to poor shielding of d and f electrons, s-electrons become inert and stability of lower oxidation states increases down the group

Statement II : The atomic size of aluminium is greater than that of gallium due to poor shielding of d-electrons.

Statement I: **In group 13, the stability of +1 oxidation state increases down the group.**

- This is **true**.
- Due to the **inert pair effect**, heavier elements like **Tl** show increased stability in the +1 oxidation state compared to lighter elements (like B and Al), which mostly prefer +3.
- So, stability of +1 state: **B < Al < Ga < In < Tl** (increases down the group).

Statement II: **The atomic size of gallium is greater than that of aluminium.**

- This is **incorrect**.
- Due to **lanthanide contraction, gallium's atomic size is actually slightly smaller or very close to aluminium's atomic size**, despite being below aluminium in the group.
- The expected trend (increasing size down the group) is **disturbed** here.

Correct option: **d) Statement I is correct but Statement II is incorrect**

45	**(c)** Graphite has a two dimensional sheet like structure. These various sheets are held together by van der Waals force of attraction. Due to these weak forces of attraction, one layer can slip over the other. Which makes graphite soft and a good lubricating agent. Due to strong $C - C$ covalent bonds, graphite is extremely difficult to melt.

About Graphite:

- Graphite has a layered structure.
- Each layer is made of carbon atoms arranged in hexagonal rings.

- Within each layer, carbon atoms are bonded strongly by covalent bonds (sp² hybridized).
- Between layers, the bonds are weak van der Waals forces, allowing layers to slide easily — this is why graphite is a good solid lubricant.
- Despite being soft due to easy sliding of layers, melting is difficult because breaking strong covalent bonds inside layers requires a lot of energy.

Evaluate options:

a) An allotropic form of diamond

- Incorrect; graphite and diamond are allotropes but graphite is not an allotrope of diamond.
- They are both allotropes of carbon, but the statement is misleading.

b) Has molecules of variable molecular masses like polymers

- Incorrect; graphite is not made of discrete molecules, it is a network solid.

c) Has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds
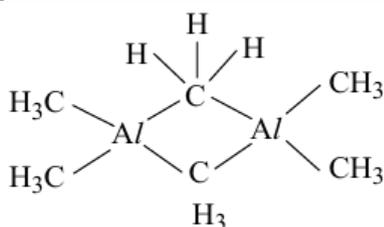
- Correct; matches the layered structure with strong covalent bonds in layers and weak forces between layers.

d) A non-crystalline substance

- Incorrect; graphite is crystalline with a well-defined layered hexagonal structure.

Correct answer: c) Has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds

46	

Coordination number of bridged carbon is 5.

Structure details:

- Trialkyl aluminium dimers, e.g., ($Al_2(CH_3)_6$), have two aluminium atoms bridged by two methyl groups.
- Each bridging methyl carbon forms 3-center-2-electron (3c-2e) bonds with two aluminium atoms.
- The carbon is bonded to:
  - Two Al atoms (bridging bonds)
  - Three hydrogens (part of the methyl group)

Coordination number:

- Coordination number counts the number of atoms directly bonded to the atom of interest (carbon in this case).
- For the bridging carbon:
  - Bonded to 2 Al atoms (bridging)
  - Bonded to 3 H atoms (its own methyl hydrogens)
- Total atoms bonded to carbon = 2 Al + 3 H = 5

So,

Coordination number of bridged carbon = 5

Summary:

- The bridged carbon is pentacoordinate (bonded to 5 atoms).
- The 3c-2e bond involves sharing electrons between 2 Al atoms and 1 carbon.
- The carbon retains its usual bonds to hydrogens as well.

Final answer: The coordination number of bridged carbon in trialkyl aluminium dimers is 5.

47

In diamond the tetrahedral carbon atoms are fused in hexagonal rings.

Diamond structure:

- Diamond consists of **carbon atoms tetrahedrally bonded** to four other carbon atoms via **sp³ hybridization**.
- The structure is a **3D network solid** formed by repeating units of tetrahedrons.
- The carbon atoms in diamond form **rings** of certain sizes — these are called **homocyclic rings** (rings containing only carbon atoms).

Number of carbon atoms in each ring:

- In diamond, the smallest rings formed are **6-membered rings**.
- These rings are **not planar** but puckered because of the 3D tetrahedral bonding.
- The diamond lattice consists of **6-membered rings** connected in a 3D network

Final answer: **The number of carbon atoms in each ring in diamond is 6.**

48
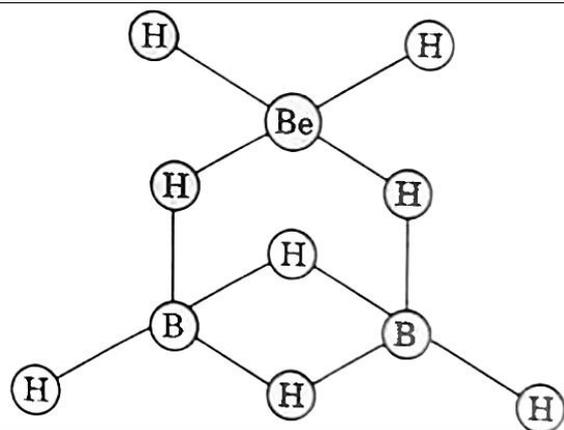
Background:

- $Be(BH_4)_2$ is known to form dimeric or polymeric structures where BH_4^- groups bridge between Be atoms.
- The bridging involves 3c-2e bonds with hydrogen atoms acting as bridges between two boron atoms or between boron and beryllium.
- Each bridging hydrogen forms a 3-center-2-electron bond.

Structure of $Be(BH_4)_2$:

- Be is coordinated by tetrahydridoborate (BH_4^-) groups.
- Typically, $Be(BH_4)_2$ exists as a dimer with two BH_4 groups bridging between two Be atoms.

	• In the dimer, each BH₄ group shares hydrogens that act as bridges. Counting 3c-2e bonds: • Each bridging BH_4^- contributes two bridging hydrogens, each forming one 3c-2e bond. • Since there are two bridging BH₄ groups, total number of 3c-2e bonds = 4. Final answer: The number of 3c-2e bonds (hydrogen bridges) in Be(BH₄)₂ is 4.
49	Each boric acid contain three OH groups which can participate in two hydrogen bonds. Structure of boric acid: • Boric acid, B(OH)₃, has three hydroxyl (–OH) groups attached to boron. • The molecule is planar, with each –OH group capable of hydrogen bonding. Hydrogen bonding in boric acid: • Each hydroxyl (–OH) group has one hydrogen atom capable of forming a hydrogen bond as a donor. • Each oxygen atom in –OH can also act as a hydrogen bond acceptor (lone pairs). • Importantly, in solid-state boric acid, molecules form cyclic trimers linked by hydrogen bonds. Number of hydrogen bonds per boric acid molecule: • Each molecule has 3 –OH hydrogens (donors). • Each molecule has 3 oxygen atoms (acceptors). • Each molecule can form up to 6 hydrogen bonds: 3 as donor + 3 as acceptor. Final answer: Each boric acid molecule can form 6 hydrogen bonds in total.
50	(3) $\quad KK(\sigma 2s)^2 (\overset{*}{\sigma} 2s)^2 (\pi 2p_x)^2 (\pi 2p_y)^2 (\sigma 2p_z)^2$ Bond order $= \dfrac{10-4}{2} = 3$
51	(b) Given expansion can be written as $$= \lim_{n \to \infty} \frac{\displaystyle\sum_{r=1}^{n-1} (r^2 - r)(n - r)}{\displaystyle\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2}$$ $$= \lim_{n \to \infty} \frac{\displaystyle\sum_{r=1}^{n-1} (-r^3 + r^2(n+1) - nr)}{\left(\dfrac{n(n+1)}{2}\right)^2 - \dfrac{n(n+1)(2n+1)}{6}}$$ $$\frac{-n^2(n-1)^2}{4} + \frac{(n+1)(n-1)n(2n-1)}{6}$$

$$= \lim_{n \to \infty} \frac{-\dfrac{n^2}{2}(n-1)}{\dfrac{n(n+1)}{2}\left(\dfrac{n(n+1)}{2} - \dfrac{2n+1}{3}\right)}$$

$$= \lim_{n \to \infty} \frac{\dfrac{-3n(n-1)^2 + 2(n+1)(n-1)(2n-1) - 6n(n-1)}{12}}{\dfrac{(n+1)}{2}\dfrac{[3n(n+1) - 2(2n+1)]}{6}}$$

$$= \lim_{n \to \infty} \frac{(n-1)\left[\begin{array}{c}-3n(n-1)\\ +2(2n^2 + 2n - n - 1) - 6n\end{array}\right]}{(n+1)[3n^2 + 3n - 4n - 2]}$$

$$= \lim_{n \to \infty} \frac{(n-1)\left[\begin{array}{c}-3n^2 + 3n + 4n^2\\ +2n - 2 - 6n\end{array}\right]}{(n+1)(3n^2 - n - 2)}$$

$$= \lim_{n \to \infty} \frac{(n-1)[n^2 - n - 2]}{(n+1)[3n^2 - n - 2]} = \frac{1}{3}$$

52	(c) Let $L = \lim\limits_{x \to \infty}$

$$\frac{(\sqrt{3x+1} + \sqrt{3x-1})^6 + (\sqrt{3x+1} - \sqrt{3x-1})^6}{(x + \sqrt{x^2-1})^6 + (x - \sqrt{x^2-1})^6} x^3$$

Now, we know that $(a+b)^6 + (a-b)^6$

$$= 2\,({}^6C_0 a^6 + {}^6C_2 a^4 b^2 + {}^6C_4 a^2 b^4 + {}^6C_6 b^6)$$

$$\therefore L = \lim_{x \to \infty} x^3$$

$$\left[\frac{2[{}^6C_0(3x+1)^3 + {}^6C_2(3x+1)^2(3x-1) + {}^6C_4(3x+1)(3x-1)^2 + {}^6C_6(3x-1)^3]}{2[{}^6C_0 x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + {}^6C_6(x^2-1)^3]}\right]$$

$$= \lim_{x \to \infty}$$

$$\frac{x^6\left[\left(3+\dfrac{1}{x}\right)^3 + 15\left(3+\dfrac{1}{x}\right)^2\left(3-\dfrac{1}{x}\right) + 15\left(3+\dfrac{1}{x}\right)\left(3-\dfrac{1}{x}\right)^2 + \left(3-\dfrac{1}{x}\right)^3\right]}{x^6\left[1 + 15\left(1-\dfrac{1}{x^2}\right) + 15\left(1-\dfrac{1}{x^2}\right)^2 + \left(1-\dfrac{1}{x^2}\right)^3\right]}$$

$$= \lim_{x \to \infty} \frac{\left[\left(3+\frac{1}{x}\right)^3 + 15\left(3+\frac{1}{x}\right)^2\left(3-\frac{1}{x}\right) + 15\left(3+\frac{1}{x}\right)\left(3-\frac{1}{x}\right)^2 + \left(3-\frac{1}{x}\right)^3\right]}{\left[1 + 15\left(1-\frac{1}{x^2}\right) + 15\left(1-\frac{1}{x^2}\right)^2 + \left(1-\frac{1}{x^2}\right)^3\right]}$$

$$= \frac{(3)^3 + 15(3)^2(3) + 15(3)(3)^2 + (3)^3}{1 + 15 + 15 + 1}$$

$$= \frac{27 + 405 + 405 + 27}{32} = \frac{864}{32} = 27$$

53

(a) Given that $f(x) = \begin{cases} \dfrac{x}{|x|} & x \neq 0 \\ 1 & x = 0 \end{cases}$,

$g(x) = \begin{cases} \dfrac{\sin(x+1)}{(x+1)} & x \neq -1 \\ 1 & x = -1 \end{cases}$ and

$h(x) = 2[x] - f(x)$

We have to find, $\lim_{x \to 1} g(h(x-1))$

Now, LHS $= \lim_{x \to 1^-} = \lim_{x \to 1^-} g(h(0^-))$

$= \lim_{x \to 1^-} g(2[0^-] - f(0^-))$

$= \lim_{x \to 1^-} g\left(2(-1) - \dfrac{0^-}{|0^-|}\right)$

$= \lim_{x \to 1^-} g(-2+1) = \lim_{x \to 1^-} g(-1) = 1$

RHS $= \lim_{x \to 1^+} g(h(x-1))$

$= \lim_{x \to 1^+} g(h(1^+ - 1)) = \lim_{x \to 1^+} g(h(0^+))$

$= \lim_{x \to 1^+} g(2[0^+] - f(0^+))$

$= \lim_{x \to 1^+} g\left(2 \times 0 - \dfrac{0^+}{|0^+|}\right)$

$= \lim_{x \to 1^+} g(-1) = 1$

LHS $=$ RHS

So, $\lim_{x \to 1} g(h(x-1)) = 1$

54

(d) At $x = 0$, RHL

$$= \lim_{x \to 0^+} \frac{\tan(\pi\sin^2 x) + (|x| - \sin(x[x]))^2}{x^2}$$

$$= \lim_{x \to 0^+} \frac{\tan(\pi\sin^2 x) + (x - \sin(x \cdot 0))^2}{x^2}.$$

$$\left[\begin{array}{l} \because |x| = x \text{ for } x > 0 \\ \text{and } [x] = 0 \text{ for } 0 < x < 1 \end{array}\right]$$

$$= \lim_{x \to 0^+} \frac{\tan(\pi\sin^2 x) + x^2}{x^2}$$

$$= \lim_{x \to 0^+} \left(\frac{\tan(\pi\sin^2 x)}{\pi\sin^2 x} \cdot \frac{\pi\sin^2 x}{x^2} + 1 \right)$$

$$= \pi \lim_{x \to 0^+} \frac{\tan(\pi\sin^2 x)}{\pi\sin^2 x} \cdot \lim_{x \to 0^+} \frac{\sin^2 x}{x^2} + 1$$

$$= \pi + 1 \qquad \left[\begin{array}{l} \because \lim_{x \to 0} \frac{\tan x}{x} = 1 \\ \text{and } \lim_{x \to 0} \frac{\sin x}{x} = 1 \end{array}\right]$$

and LHL

$$= \lim_{x \to 0^-} \frac{\tan(\pi\sin^2 x) + (|x| - \sin(x[x])^2}{x^2}$$

$$= \lim_{x \to 0^-} \frac{\tan(\pi\sin^2 x) + (-x - \sin(x(-1))^2}{x^2}$$

$$\left[\begin{array}{l} \because |x| = -x \text{ for } x < 0 \\ \text{and } [x] = -1 \text{ for } -1 < x < 0 \end{array}\right]$$

$$= \lim_{x \to 0^-} \frac{\tan(\pi\sin^2 x) + (x + \sin(-x))^2}{x^2}$$

$$= \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x) + (x - \sin x)^2}{x^2}$$

$$[\because \sin(-\theta) = -\sin \theta]$$

$$= \lim_{x \to 0^-} \left(\frac{\tan(\pi \sin^2 x) + x^2 + \sin^2 x - 2x \sin x}{x^2} \right)$$

$$= \lim_{x \to 0^-} \left(\frac{\tan(\pi \sin^2 x)}{x^2} + 1 + \frac{\sin^2 x}{x^2} - \frac{2x \sin x}{x^2} \right)$$

$$= \lim_{x \to 0^-} \left(\frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \frac{\pi \sin^2 x}{x^2} + 1 \right.$$

$$\left. + \frac{\sin^2 x}{x^2} - 2 \frac{\sin x}{x} \right)$$

$$= \lim_{x \to 0^-} \frac{\tan(\pi \sin^2 x)}{\pi \sin^2 x} \cdot \lim_{x \to 0^-} \frac{\pi \sin^2 x}{x^2} +$$

$$1 + \lim_{x \to 0^-} \frac{\sin^2 x}{x^2} - 2 \lim_{x \to 0^-} \frac{\sin x}{x}$$

$$= \pi + 1 + 1 - 2 = \pi$$

$\because$ RHL $\neq$ LHL

$\therefore$ Limit does not exist.

55	(a) Given,

$$\lim_{x \to 1^+} \frac{(1 - |x| + \sin|1 - x|) \sin\left(\frac{\pi}{2} [1 - x] \right)}{|1 - x| [1 - x]}$$

Let $x = 1 + h$, then $x \to 1^+ \Rightarrow h \to 0^+$

$$\therefore \lim_{x \to 1^+} \frac{(1 - |x| + \sin|1 - x|) \sin\left(\frac{\pi}{2} [1 - x] \right)}{|1 - x| [1 - x]}$$

$$(1 - |h + 1| + \sin|-h|)$$

$$= \lim_{h \to 0^+} \frac{\sin\left(\frac{\pi}{2}[-h]\right)}{|-h|[-h]}$$

$$= \lim_{h \to 0^+} \frac{(1-(h+1)+\sin h)\sin\left(\frac{\pi}{2}[-h]\right)}{h[-h]}$$

$$[\because |-h| = h \text{ and } |h+1| = h+1 \text{ as } h > 0]$$

$$= \lim_{h \to 0^+} \frac{(-h+\sin h)\sin\left(\frac{\pi}{2}(-1)\right)}{h(-1)}$$

$$(\because [x] = -1 \text{ for } -1 < x < 0 \text{ and } h \to 0^+$$

$$\Rightarrow -h \to 0^-)$$

$$= \lim_{h \to 0^+} \frac{(-h+\sin h)}{-h}\sin\left(\frac{-\pi}{2}\right)$$

$$= \lim_{h \to 0^+} \left(\frac{\sin h}{h}\right) - \lim_{h \to 0^+} \left(\frac{h}{h}\right)$$

$$= \lim_{h \to 0^+} \left(\frac{\sin h}{h}\right) - \lim_{h \to 0^+} \left(\frac{h}{h}\right)$$

$$= 1 - 1 = 0 \qquad \left[\because \lim_{h \to 0^+} \frac{\sin h}{h} = 1\right]$$

56

(b) $\displaystyle \lim_{x \to 0} = \frac{e - (1+2x)^{1/2x}}{x}$

$$= \lim_{x \to 0} = \frac{e - e^{\frac{1}{2x}\ln(1+2x)}}{x}$$

$$= \lim_{x \to 0}(-e)\frac{(e^{\frac{\ln(1+2x)}{2x}-1}-1)}{x}$$

$$= \lim_{x \to 0}(-e)\frac{e^{\frac{\ln(1+2x)}{2x}-1}-1}{\frac{\ln(1+2x)}{2x}-1},$$

$$= \lim_{x \to 0} (-e) \dfrac{\dfrac{\ln(1+2x)-1}{2x}}{x}$$

$$= \lim_{x \to 0} (-e) \dfrac{\ln(1+2x)-2x}{2x^2}$$

$$= \lim_{x \to 0} (-e) \dfrac{\left[\left(2x - \dfrac{(2x)^2}{2} + \ldots\right) - 2x\right]}{2x^2}$$

$$= e \times 1 = e$$

---

57 (a) We have,

$$\lim_{x \to \alpha} \left([x-5] - [2x+2]\right) = 0$$
$$[\alpha - 5] - [2\alpha + 2] = 0$$
$$[\alpha - 5] = [2\alpha + 2] \quad \ldots(i)$$

For algebraic calculation,

$$\alpha - 5 = 2\alpha + 2 \Rightarrow \alpha = -7$$

So, its value will be around -7.

For $\alpha = -7.5$ in Eq. (i),

$$[-7.5 - 5] = [-15 + 2]$$
$$\Rightarrow \quad [-12.5] = [-13]$$
$$\Rightarrow \quad -13 = -13 \Rightarrow \text{True}$$

For $\alpha = -6.5$ in Eq. (i),

$$[-6.5 - 5] = [-13 + 2]$$
$$\Rightarrow \quad [-11.5] = [-11]$$
$$\Rightarrow \quad -12 \neq -11$$
$$\Rightarrow \text{Not satisfies.}$$

$\therefore$ The required range will be

$$\alpha \in [-7.5, -6.5).$$

---

58 (c) Using binomial

$$\lim_{n \to \infty} n\left(1 - \dfrac{n+1}{n^2}\right)^{\frac{1}{2}} + \alpha n + \beta = 0$$

$$\lim_{n \to \infty} n\left\{1 - \dfrac{1}{2}\left(\dfrac{n+1}{n^2}\right)\right.$$

$$\left. + \dfrac{\left(\dfrac{1}{2}\right)\left(-\dfrac{1}{2}\right)}{2!}\left(\dfrac{n+1}{n^2}\right)^2 + \ldots\right\}$$

$$+ \alpha n + \beta = 0$$

$$\lim_{n \to \infty} \left(n - \dfrac{1}{2} + \dfrac{1}{n} + \ldots + n\alpha + \beta\right) = 0$$

$$\lim_{n \to \infty} \left[(\alpha + 1)n + \left(\beta - \dfrac{1}{2}\right) + \dfrac{1}{n} + \ldots\right]$$

$$\alpha = -1, \beta = \frac{1}{2} \implies 8(\alpha + \beta) = -4$$

59

(a) Clearly, $\lim\limits_{y \to 0} \dfrac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$

$$= \lim_{y \to 0} \left(\frac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4} \right.$$

$$\left. \times \frac{\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2}}{\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2}} \right)$$

[rationalising the numerator]

$$= \lim_{y \to 0} \frac{(1 + \sqrt{1 + y^4}) - 2}{y^4 (\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2})}$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$= \lim_{y \to 0} \left(\frac{\sqrt{1 + y^4} - 1}{y^4 (\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2})} \right.$$

$$\left. \times \frac{\sqrt{1 + y^4} + 1}{\sqrt{1 + y^4} + 1} \right)$$

[again, rationalising the numerator]

$$= \lim_{y \to 0} \frac{y^4}{y^4 (\sqrt{1 + \sqrt{1 + y^4}} + \sqrt{2})(\sqrt{1 + y^4} + 1)}$$

$$= \frac{1}{2\sqrt{2} \times 2}$$

(by cancelling y^4 and then by direct substitution).

$$= 1/4\sqrt{2}$$

60

(b) α, β are the roots of the equation

$$ax^2 + bx + 1 = 0$$

⇒ The roots of the equation

$x^2 + bx + a = 0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

$$\Rightarrow \quad x^2 + bx + a = \left(x - \frac{1}{\alpha}\right)\left(x - \frac{1}{\beta}\right)$$

∴ The given limit

$$= \lim_{x \to \frac{1}{\alpha}} \left[\frac{1 - \cos\left(x - \dfrac{1}{\alpha}\right)\left(x - \dfrac{1}{\beta}\right)}{2\alpha^2\left(x - \dfrac{1}{\alpha}\right)^2}\right]^{1/2}$$

$$= \lim_{x \to \frac{1}{\alpha}} \left[\frac{1 - \cos\left(x - \dfrac{1}{\alpha}\right)\left(x - \dfrac{1}{\beta}\right)}{2\alpha^2\left(x - \dfrac{1}{\alpha}\right)^2\left(x - \dfrac{1}{\beta}\right)^2}\right]^{1/2}$$

$$\times\left(x - \frac{1}{\beta}\right)$$

$$= \lim_{x \to \frac{1}{\alpha}} \left[\frac{2\sin^2\left(\dfrac{\left(x - \dfrac{1}{\alpha}\right)\left(x - \dfrac{1}{\beta}\right)}{2}\right)}{2\alpha^2\left\{\dfrac{\left(x - \dfrac{1}{\alpha}\right)\left(x - \dfrac{1}{\beta}\right)}{2}\right\}^2 \times 4}\right]^{1/2}$$

$$\times\left(x - \frac{1}{\beta}\right) = \frac{1}{2\alpha}\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)$$

∴ $k = 2\alpha$

61

(c) $\displaystyle\lim_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$

$= \displaystyle\lim_{x \to 0} \frac{2\sin\left(\dfrac{\sin x + x}{2}\right)\cdot\sin\left(\dfrac{x - \sin x}{2}\right)}{x^4}$

$$\left[\frac{0}{0} \text{ form}\right]$$

$= \displaystyle\lim_{x \to 0} \frac{2\sin\left(\dfrac{\sin x + x}{2}\right)}{\left(\dfrac{\sin x + x}{2}\right)} \cdot \frac{\sin\left(\dfrac{x - \sin x}{2}\right)}{\left(\dfrac{x - \sin x}{2}\right)}$

$$\times \left(\frac{x + \sin x}{2}\right) \times \left(\frac{x - \sin x}{2}\right)\frac{1}{x^4}$$

$= \displaystyle\lim_{x \to 0} 2\left[\frac{\sin\left(\dfrac{\sin x + x}{2}\right)}{\left(\dfrac{\sin x + x}{2}\right)}\right]\left[\frac{\sin\left(\dfrac{x - \sin x}{2}\right)}{\left(\dfrac{x - \sin x}{2}\right)}\right]$

$$\left(\frac{x^2 - \sin^2 x}{4}\right)\cdot\frac{1}{x^4}$$

We know that, $\displaystyle\lim_{t \to 0} \frac{\sin t}{t} = 1$

$\therefore \qquad \displaystyle\lim_{x \to 0} 2\left(\frac{x^2 - \sin^2 x}{4x^4}\right) \qquad \left[\frac{0}{0}\text{ form}\right]$

Using L' Hospital's rule

$\displaystyle\lim_{x \to 0} 2\left(\frac{2x - 2\sin x\cos x}{16x^3}\right)$

$= \displaystyle\lim_{x \to 0} \frac{2x - \sin 2x}{8x^3} \qquad \left[\frac{0}{0}\text{ form}\right]$

Using L' Hospital's rule,

$\displaystyle\lim_{x \to 0} \frac{2 - 2\cos 2x}{24x^2} \qquad \left[\frac{0}{0}\text{ form}\right]$

$= \displaystyle\lim_{x \to 0}\left(\frac{4\sin 2x}{48x}\right)$

$= \dfrac{1}{6}\displaystyle\lim_{x \to 0}\cdot\frac{\sin 2x}{2x} \qquad \left[\displaystyle\lim_{x \to 0}\frac{\sin t}{t} = 1\right]$

$\therefore \qquad = 1/6$

62 (c) Given α and β are roots of quadratic equation $375x^2 - 25x - 2 = 0$

$$\therefore \quad \alpha + \beta = \frac{25}{375} = \frac{1}{15} \quad \ldots \text{(i)}$$

and

$$\alpha\beta = -\frac{2}{375} \quad \ldots \text{(ii)}$$

Now, $\displaystyle \lim_{n \to \infty} \sum_{r=1}^{n} \alpha^r + \lim_{n \to \infty} \sum_{r=1}^{n} \beta^r$

$= (\alpha + \alpha^2 + \alpha^3 + \ldots + \text{upto infinite terms}) + (\beta + \beta^2 + \beta^3 + \ldots + \text{upto infinite terms})$

$$= \frac{\alpha}{1 - \alpha} + \frac{\beta}{1 - \beta} \quad \left[\because S_\infty = \frac{a}{1 - r} \text{ for GP} \right]$$

$$= \frac{\alpha(1 - \beta) + \beta(1 - \alpha)}{(1 - \alpha)(1 - \beta)}$$

$$= \frac{\alpha - \alpha\beta + \beta - \alpha\beta}{1 - \alpha - \beta + \alpha\beta}$$

$$= \frac{(\alpha + \beta) - 2\alpha\beta}{1 - (\alpha + \beta) + \alpha\beta}$$

On substituting the value $\alpha + \beta = \dfrac{1}{15}$ and

$\alpha\beta = \dfrac{-2}{375}$ from Eqs. (i) and (ii)

respectively, we get

$$\frac{\dfrac{1}{15} + \dfrac{4}{375}}{1 - \dfrac{1}{15} - \dfrac{2}{375}} = \frac{29}{375 - 25 - 2} = \frac{29}{348} = \frac{1}{12}$$

63

(d) Since, $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

$\Rightarrow \quad f(x) = \begin{cases} \dfrac{\sin[x]}{[x]}, & x \in R - [0, 1) \\ 0, & 0 \leq x < 1 \end{cases}$

At $x = 0$, RHL $= \lim\limits_{x \to 0^+} 0 = 0$

and   LHL

$= \lim\limits_{x \to 0^-} \dfrac{\sin[x]}{[x]} = \lim\limits_{h \to 0} \dfrac{\sin[0-h]}{[0-h]}$

$= \lim\limits_{h \to 0} \dfrac{\sin(-1)}{-1} = \sin 1$

Since, RHL $\neq$ LHL

$\therefore$ Limit does not exist.

64

(d) LHL $= \lim\limits_{x \to 1^-} \dfrac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

$= \lim\limits_{x \to 1^-} \dfrac{\sqrt{2\sin^2(x-1)}}{x-1}$

$= \sqrt{2} \lim\limits_{x \to 1^-} \dfrac{|\sin(x-1)|}{x-1}$

Let $x = 1 - h, h > 0$,

for $x \to 1^-, h \to 0$

$= \sqrt{2} \lim\limits_{h \to 0} \dfrac{|\sin(-h)|}{-h}$

$= \sqrt{2} \lim\limits_{h \to 0} \dfrac{\sin h}{-h} = -\sqrt{2}$

Again, RHL $= \lim\limits_{x \to 1^+} \dfrac{\sqrt{1 - \cos 2(x-1)}}{x-1}$

$= \lim\limits_{x \to 1^+} \sqrt{2} \cdot \dfrac{|\sin(x-1)|}{x-1}$

Let $x = 1 + h, h > 0$

For $x \to 1^+, h \to 0$

$= \lim\limits_{h \to 0} \sqrt{2} \dfrac{|\sin h|}{h}$

$= \lim\limits_{h \to 0} \sqrt{2} \dfrac{\sin h}{h} = \sqrt{2}$

$\therefore$ LHL $\neq$ RHL.

Hence, $\lim\limits_{x \to 1} f(x)$ does not exist.

65 (d) $x = 2$ is a root of $x^2 + px + q = 0$

$\Rightarrow \quad 2^2 + 2p + q = 0$

$\Rightarrow \quad q + 4 = -2p \qquad \ldots(i)$

$\therefore q^2 + 8q + 16 = (q + 4)^2 = 4p^2 \quad \ldots(ii)$

and $x^2 - 4px + (q^2 + 8q + 16)$

$= x^2 - 4px + 4p^2 = (x - 2p)^2 \quad \ldots(iii)$

We have, $f(x) =$

$$\begin{cases} \dfrac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x - 2p)^4}, & x \neq 2p \\ \quad 0 & x = 2p \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \dfrac{1 - \cos(x - 2p)^2}{(x - 2p)^4}, & x \neq 2p \\ \quad 0, & x = 2p \end{cases}$$

Now, as $x \to 2p^+$, $x - 2p \to 0^+$

$$\therefore \lim_{x \to 2p^+} f(x) = \frac{1}{2}$$

$$\left[\because \lim_{y \to 0} \frac{1 - \cos y}{y^2} = \frac{1}{2} \right]$$

Hence, $\lim\limits_{x \to 2p^+} [f(x)] = 0$, where $[\cdot]$ is

the greatest integer function.

---

66 (d) $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sin\left(\sin^{-1}\sqrt{1 - x^2}\right) - x}{1 - \tan\left(\tan^{-1}\left(\dfrac{\sqrt{1 - x^2}}{x}\right)\right)}$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{\sqrt{1 - x^2} - x}{1 - \left(\dfrac{\sqrt{1 - x^2}}{x}\right)}$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} \dfrac{(\sqrt{1 - x^2} - x)}{(\sqrt{1 - x^2} - x)} \times (-x)$

$= \lim\limits_{x \to \frac{1}{\sqrt{2}}} (-x) = -\left(\dfrac{1}{\sqrt{2}}\right)$

67	*(d)* Here, $\lim\limits_{x \to 0} \{1 + x \log(1 + b^2)\}^{1/x}$

$\qquad\qquad\qquad\qquad$ [1^∞ form]

$= e^{\lim\limits_{x \to 0} \{x \log(1 + b^2)\} \cdot \frac{1}{x}}$

$= e^{\log(1 + b^2)} = (1 + b^2)$ $\qquad$ …(i)

Given, $\lim\limits_{x \to 0} \{1 + x \log(1 + b^2)\}^{1/x}$

$\qquad\qquad = 2b \sin^2\theta$

$\Rightarrow (1 + b^2) = 2b \sin^2\theta$

$\therefore \quad \sin^2\theta = \dfrac{1 + b^2}{2b}$ $\qquad$ …(ii)

By AM $\geq$ GM, $\dfrac{b + \dfrac{1}{b}}{2} \geq \left(b \cdot \dfrac{1}{b}\right)^{1/2}$

$\Rightarrow \qquad\qquad \dfrac{b^2 + 1}{2b} \geq 1$ $\qquad$ …(iii)

From Eqs. (ii) and (iii),

$\qquad\qquad \sin^2\theta = 1$

$\Rightarrow \qquad\qquad \theta = \pm\dfrac{\pi}{2}$, as $\theta \in (-\pi, \pi]$ |
| **68** | **Answer: (a) Both true, and R explains A.**

- $1 - \cos 2(x - \alpha) = 2\sin^2(x - \alpha) \Rightarrow \dfrac{\sqrt{1 - \cos 2(x - \alpha)}}{x - \alpha} = \sqrt{2}\,\dfrac{|\sin h|}{h}$ with $h \to 0$.

  Right/left limits: $\sqrt{2}$ and $-\sqrt{2} \Rightarrow$ does not exist $\Rightarrow$ A true.

- $\lim\limits_{x \to 0} \dfrac{|\sin x|}{x}$: right $= 1$, left $= -1 \Rightarrow$ does not exist $\Rightarrow$ R true. |
| **69** | **Answer: D**

For $x \to 1^+$: let $x = 1 + h, h > 0$.

$|x - 1| = |h| = h, \quad |x| = 1 + h$.

So inside $= h + (1 + h) = 1 + 2h$.

As $h \to 0^+$, inside $\to 1^+$.

Then $\lfloor 1 + 2h \rfloor = 1$.

$\Rightarrow$ RHL = 1.

For $x \to 1^-$: let $x = 1 - h, h > 0$.

$|x - 1| = |-h| = h, \quad |x| = |1 - h| = 1 - h$ (since $1 - h > 0$ for small h).

So inside $= h + (1 - h) = 1$.

Then $\lfloor 1 \rfloor = 1$.

$\Rightarrow$ LHL = 1.

Since the RHL (1) is equal to the LHL (1), the limit exists and is equal to 1.

Therefore, $\lim\limits_{x \to 1}[|x - 1| + |x|] = 1$.

The Assertion (A) is false, and the Reason (R) is true. |

70 | **Answer: B**

A) $\lim\limits_{x \to 5} \dfrac{x^2 - 9x + 20}{x - \lfloor x \rfloor}$

   Here $\lfloor x \rfloor$ is the greatest integer $\leq x$. Factor the numerator:

   $$x^2 - 9x + 20 = (x - 4)(x - 5).$$

   - As $x \to 5^-$, $\lfloor x \rfloor = 4$. Then

   $$\frac{(x - 4)(x - 5)}{x - 4} = x - 5 \implies \lim_{x \to 5^-} = 0.$$

   - As $x \to 5^+$, $\lfloor x \rfloor = 5$. Then

   $$\frac{(x - 4)(x - 5)}{x - 5} = x - 4 \implies \lim_{x \to 5^+} = 1.$$

   Since LHL $= 0$ and RHL $= 1$, the two-sided limit **does not exist**.

B) $\lim\limits_{n \to \infty} \dfrac{\lfloor x \rfloor + \lfloor 2x \rfloor + \cdots + \lfloor nx \rfloor}{n^2}$

   Write $\lfloor kx \rfloor = kx - \{kx\}$ with $0 \leq \{kx\} < 1$. Then

   $$\sum_{k=1}^{n} \lfloor kx \rfloor = \sum_{k=1}^{n} kx - \sum_{k=1}^{n} \{kx\} = x\frac{n(n+1)}{2} - S_n,$$

   where $0 \leq S_n < n$.

   Divide by n^2 and let $n \to \infty$:

   $$\frac{x}{2} \cdot \frac{n(n+1)}{n^2} - \frac{S_n}{n^2} \longrightarrow \frac{x}{2} - 0 = \boxed{\frac{x}{2}}.$$

C) $\lim\limits_{x \to \frac{1}{2}^-} \dfrac{\lfloor x + 1 \rfloor}{x^2}$

   As $x \to \frac{1}{2}^-$, $x + 1 \to 1.5^- \Rightarrow \lfloor x + 1 \rfloor = 1$.
   Denominator: $x^2 \to (1/2)^2 = 1/4$.

   So limit $= \frac{1}{1/4} = 4.$ ✅

D) $\lim\limits_{x \to \frac{\pi}{2}} \lfloor \sin x \rfloor$.

   As $x \to \frac{\pi}{2}^-$: $\sin x \to 1^-$. So $\lfloor \sin x \rfloor = 0$.
   As $x \to \frac{\pi}{2}^+$: $\sin x \to 1^-$ (since $\sin(\pi/2 + h) = \cos h \approx 1 - h^2/2 < 1$).

   So again $\lfloor \sin x \rfloor = 0$.

   $$\lim_{x \to \frac{\pi}{2}} \lfloor \sin x \rfloor = 0.$$

71

$$\log\left[\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right]$$

$$= \exp\lim_{x\to 0}\left[\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right) - 1\right] \times \frac{100}{x}$$

. *(1)* Let

$$y = \lim_{x\to 10}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$$

$$[1^\infty \text{ form}]$$

Now, $\log y = \lim\limits_{x\to 10}\left(\dfrac{100}{x}\right)$

$$= \exp\lim_{x\to 0}\left[\frac{100}{x}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x) - ((x+2)^3 + 2(x+2)^2 + 3\sin(x+2))}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)\right]$$

$$= \exp \lim_{x \to 0} \frac{100}{x} \left[\left(\frac{\begin{array}{c}(x + 2\cos x)^3 \\ -(x + 2)^3 + \\ 2(x + 2\cos x)^2 \\ -2(x + 2)^2 + 3\sin \\ (x + 2\cos x) - \\ 3\sin(x + 2)\end{array}}{8 + 8 + 3\sin 2} \right) \right]$$

$$= e^{\frac{100}{16 + 3\sin 2}}$$

$$\lim_{x \to 0} \frac{\begin{array}{c}[3(x + 2\cos x)^2 \times (1 - 2\sin x) \\ -3(x + 2)^2 - 4(x + 2\cos x) \\ \times (1 - 2\sin x) - 4(x + 2) \\ + 3\cos(x + 2\cos x) \\ \times (1 - 2\sin x) - 3\cos(x - 2)]\end{array}}{1}$$

[By L′ Hospital rule]

$$= e^{\frac{100}{16 + 3\sin 2}} \left(\frac{12 - 3(4) + 8 \times 1 - 8}{1} + 3\cos 2 - 3\cos 2 \right)$$

$$e^0 = 1$$

72

(18) Finding right hand limit

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} f(h)$$

$$= \lim_{h \to 0} \frac{\cos^{-1}(1 - h^2)\sin^{-1}(1 - h)}{h(1 - h^2)}$$

$$= \lim_{h \to 0} \frac{\cos^{-1}(1 - h^2)}{h} \left(\frac{\sin^{-1} 1}{1} \right)$$

Let $\cos^{-1}(1 - h^2) = \theta$

$$\Rightarrow \quad \cos\theta = 1 - h^2 = \frac{\pi}{2} \lim_{\theta \to 0} \frac{\theta}{\sqrt{1 - \cos\theta}}$$

$$= \frac{\pi}{2} \lim_{\theta \to 0} \frac{1}{\sqrt{\frac{1 - \cos\theta}{\theta^2}}} = \frac{\pi}{2} \frac{1}{\sqrt{\frac{1}{2}}}$$

$$\therefore \quad R = \frac{\pi}{\sqrt{2}}$$

$$\therefore \quad L = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(-h)$$

$$= \lim_{h \to 0} \frac{\cos^{-1}(1 - \{-h\}^2) \sin^{-1}(1 - \{-h\})}{\{-h\} - \{-h\}^3}$$

$$= \lim_{h \to 0} \frac{\cos^{-1}(1 - (-h+1)^2) \sin^{-1}(1 - (-h+1))}{(-h+1) - (-h+1)^3}$$

$$= \lim_{h \to 0} \frac{\cos^{-1}(-h^2 + 2h) \sin^{-1} h}{(1-h)(1 - (1-h)^2)}$$

$$= \lim_{h \to 0} \left(\frac{\pi}{2}\right) \frac{\sin^{-1} h}{(1 - (1-h)^2)}$$

$$= \frac{\pi}{2} \lim_{h \to 0} \left(\frac{\sin^{-1} h}{-h^2 + 2h}\right)$$

$$= \frac{\pi}{2} \lim_{h \to 0} \left(\frac{\sin^{-1} h}{h}\right)\left(\frac{1}{-h+2}\right)$$

$$\therefore \qquad L = \pi/4$$

$$\Rightarrow \frac{32}{\pi^2}(L^2 + R^2) = \frac{32}{\pi^2}\left(\frac{\pi^2}{2} + \frac{\pi^2}{16}\right)$$

$$= 16 + 2 = 18$$

73

(8) $\displaystyle\lim_{x \to \pi/2}$

$$\frac{4\sqrt{2} \cdot 2 \sin 2x \cos x}{2\sin 2x \sin \frac{3x}{2} + \left(\cos \frac{5x}{2} - \cos \frac{3x}{2}\right)}$$
$$- \sqrt{2}(1 + \cos 2x)$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{8\sqrt{2} \cdot 2 \sin x \cos x \cos x}{2 \sin 2x \sin \frac{3x}{2} - 2 \sin 2x \sin \frac{x}{2}}$$
$$- 2\sqrt{2} \cos^2 x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{16\sqrt{2} \sin x \cos^2 x}{2 \sin 2x \left(\sin \frac{3x}{2} - \sin \frac{x}{2}\right)}$$
$$- 2\sqrt{2} \cos^2 x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{16\sqrt{2} \sin x \cos^2 x}{4 \sin x \cos x \left(2 \cos x \cdot \sin \frac{x}{2}\right)}$$
$$- 2\sqrt{2}\cos^2 x$$

$$= \frac{16\sqrt{2} \sin x \cos^2 x}{2\cos^2 x \left(4 \sin x \sin \frac{x}{2} - \sqrt{2}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{8\sqrt{2} \sin x}{4 \sin x \sin \frac{x}{2} - \sqrt{2}} = 8$$

74 . (81)

$$\lim_{x \to 0} \frac{ax^2 e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$$

$$\lim_{x \to 0} ax^2 \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \right)$$

$$-b \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots \right)$$

$$\frac{+ cx \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \right)}{x^3 \left(\frac{\sin x}{x} \right)} = 1$$

$$\Rightarrow \lim_{x \to 0} (c - b)x + \left(\frac{b}{2} - c + a \right) x^2$$

$$\frac{+ \left(a - \frac{b}{3} + \frac{c}{2} \right) x^3 + \ldots}{x^3} = 1$$

$$c - b = 0 \qquad \ldots(i)$$

$$\frac{b}{2} - c + a = 0 \qquad \ldots(ii)$$

$$a - \frac{b}{3} + \frac{c}{2} = 1 \qquad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$a = 3/4, \, b = c = 3/2$$

$$\therefore 16(a^2 + b^2 + c^2) = 16 \left(\frac{9}{16} + \frac{9}{4} + \frac{9}{4} \right)$$

$$= 9 + 72 = 81$$

75

(2) Given, $\lim\limits_{\alpha \to 0}\left[\dfrac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right] = -\dfrac{e}{2}$

$\Rightarrow \lim\limits_{\alpha \to 0}\dfrac{e\{e^{\cos(\alpha^n)-1}-1\}}{\cos(\alpha^n)-1}\cdot\dfrac{\cos(\alpha^n)-1}{\alpha^m}$

$= \dfrac{-e}{2}$

$\Rightarrow \lim\limits_{\alpha \to 0} e\left\{\dfrac{e^{\cos(\alpha^n)-1}-1}{\cos(\alpha^n)-1}\right\}$

$\cdot \lim\limits_{\alpha \to 0}\dfrac{-2\sin^2\dfrac{\alpha^n}{2}}{\alpha^m} = -e/2$

$\Rightarrow e\times 1\times(-2)\lim\limits_{\alpha \to 0}\dfrac{\sin^2\left(\dfrac{\alpha^n}{2}\right)}{\dfrac{\alpha^{2n\cdot}}{4}}\cdot\dfrac{\alpha^{2n}}{4\alpha^m} = \dfrac{-e}{2}$

$\Rightarrow e\times 1\times(-2)\times 1\times \lim\limits_{\alpha \to 0}\dfrac{\alpha^{2n-m}}{4} = \dfrac{-e}{2}$

For this to be exists, $2n - m = 0$

$\Rightarrow \qquad\qquad m/n = 2$